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	Values of n.					Values of n.			
	$n = 1$	$n = \frac{2}{3}$	$n = \frac{1}{2}$	$n = \frac{1}{3}$		$n = 1$	$n = \frac{2}{3}$	$n = \frac{1}{2}$	$n = \frac{1}{3}$
$p =$ °	$\theta =$ °	$\theta =$ °	$\theta =$ °	$\theta =$ °	$p =$ °	$\theta =$ °	$\theta =$ °	$\theta =$ °	$\theta =$ °
0	0·000	0·000	0·000	0·000	80	0·839	0·890	0·916	0·943
10	0·087	0·197	0·296	0·444	90	1·000	1·000	1·000	1·000
20	0·176	0·314	0·420	0·561	100	1·192	1·124	1·092	1·060
30	0·268	0·416	0·518	0·645	110	1·428	1·268	1·195	1·126
40	0·364	0·510	0·603	0·714	120	1·732	1·442	1·316	1·201
50	0·466	0·601	0·683	0·776	130	2·144	1·663	1·464	1·290
60	0·577	0·693	0·760	0·833	140	2·747	1·962	1·657	1·401
70	0·700	0·788	0·837	0·888	150	3·732	2·406	1·932	1·551
80	0·839	0·890	0·916	0·943	160	5·671	3·173	2·381	1·783

The first series of numbers exhibits the projection of the radii of the successive projected parallels in the stereographic projection; the second, in that which occupies a section of 240°, such as by cutting out the unoccupied portion would roll up into a cone, well adapted for a transparent map on a lamp-shade; the third, in that which occupies a semicircle (exhibited in fig. B), a convenient form for a reference chart, rejecting lateral continuations, but which becomes too much distended beyond the 55th parallel of south latitude; and the last, that comprised in a sector of 120°, as in figure A, which is preferable to either, and seems to me not unlikely to supersede all other projections for a general chart.

VIII.—*Description of the Projection used in the Topographical Department of the War Office for Maps embracing large portions of the Earth's Surface.*[*]

Communicated by **Colonel Sir Henry James**, R.E., F.R.G.S., &c.,
Director of the Topographical Department.

The method of development adopted in the Topographical Department for the construction of the maps of North and South

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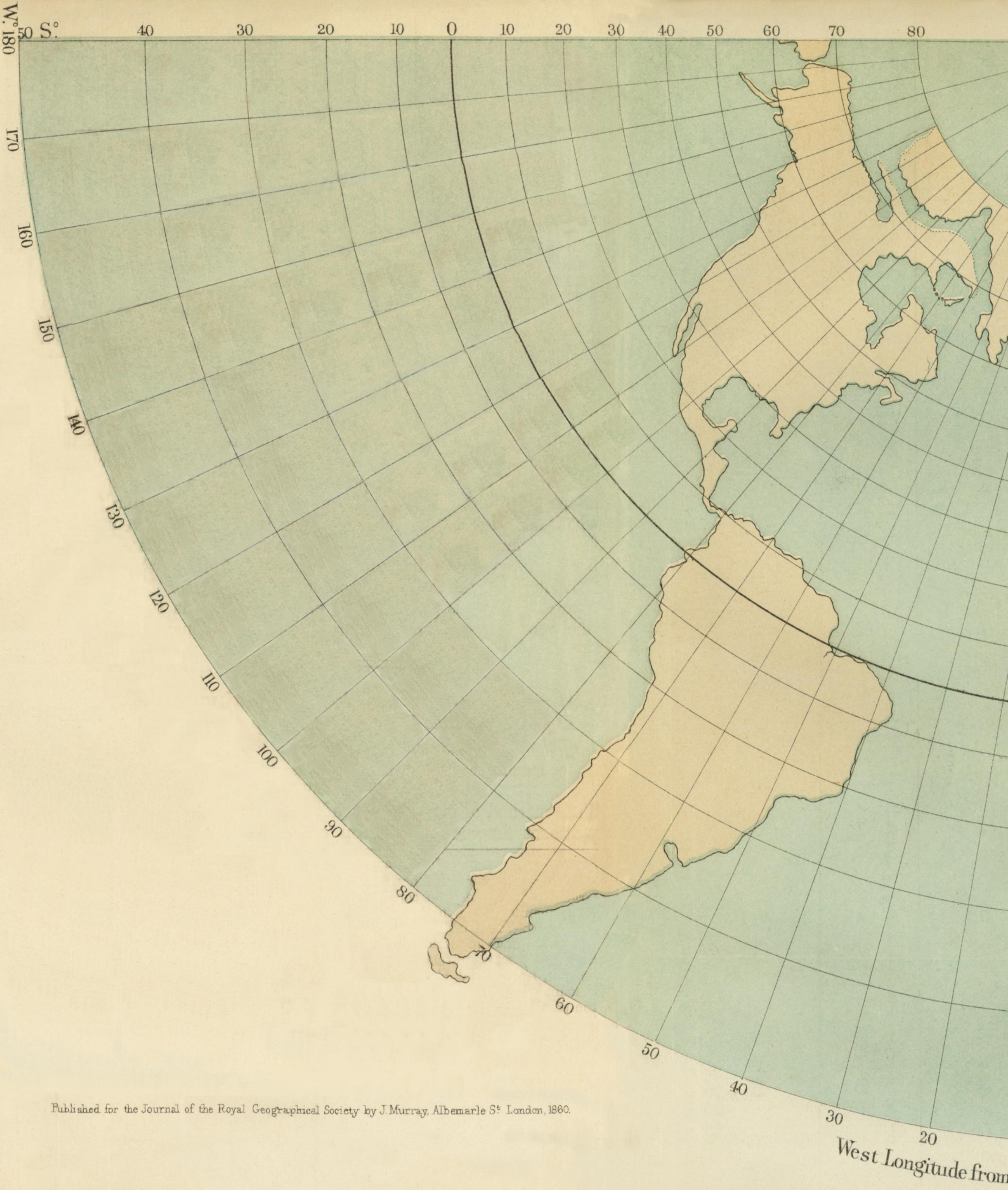
[*] Sir, Ordnance Survey Office, Southampton, 2nd July, 1858.

The projection which I have adopted for the construction of maps embracing large areas possesses, as I conceive, great advantages over any other with which I am acquainted; and thinking that a description of the mode of constructing such maps, with a table of the lengths of the radii of the arcs of parallels, and the distances between the meridians on the several parallels, which renders the construction on any scale extremely easy, would be acceptable to the Royal Geographical Society, I requested Captain Alexander Clarke, R.E., to draw up the following account of the method employed by us. Every meridian in this projection cuts the parallels at right angles, and the distortion (which is incidental to every projection) is in this diminished to a greater extent than in any other.

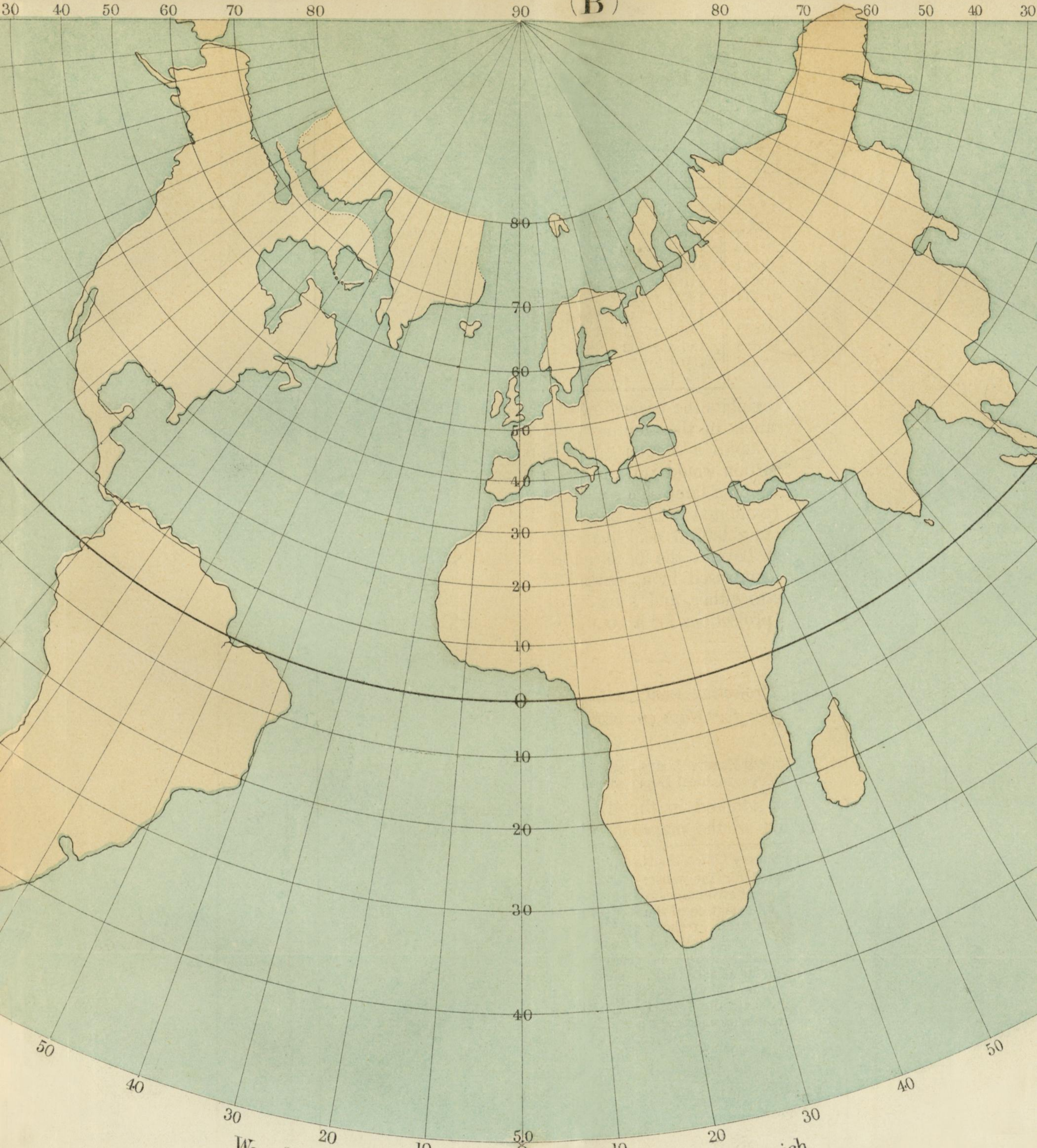
I am, Sir, your obedient servant,
Henry James, Colonel Royal Engineers,
Director Topographical Department.

Norton Shaw, Esq.,
Secretary Royal Geographical Society.

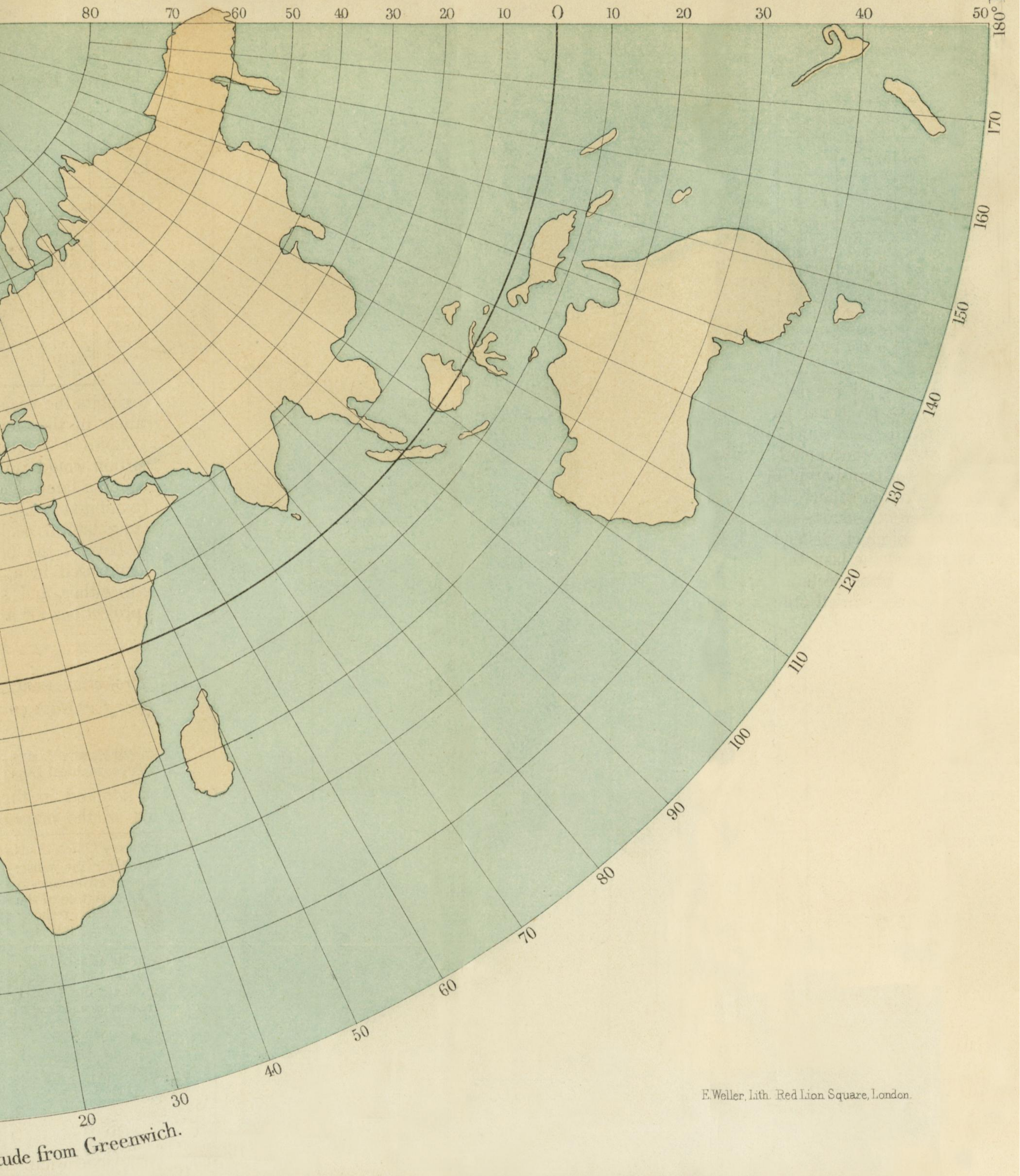


Published for the Journal of the Royal Geographical Society by J. Murray, Albemarle S^t. London, 1860.

(B)



West Longitude from Greenwich. East Longitude from Greenwich.



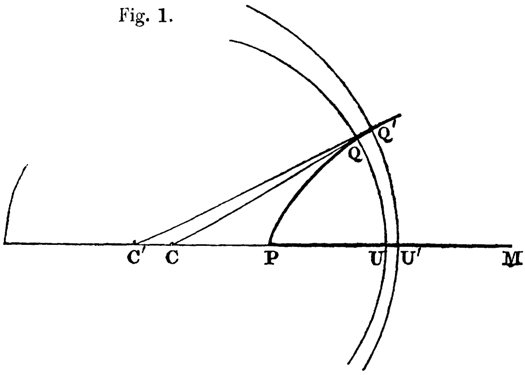
E. Weller, Lith. Red Lion Square, London.

America differs from any that has been hitherto employed, and has several advantages which seem to give it a claim to special notice.

1. Every projection or method of development, when applied to a very large tract of country, such as Asia, must necessarily exhibit distortion. If, as in the Stereographic or Mercator's Projection, we maintain the correct representation of all small portions, considered merely in themselves, and without reference to adjoining parts, we must put up with considerable distortion of areas or of the linear scale; and if, on the other hand, we endeavour to avoid this misrepresentation of area, we must have a distortion of form in the small portions—squares on the actual surface being developed into parallelograms, often very oblique. In the system of development which forms the subject of this paper, every square on the actual surface having its sides parallel and perpendicular to the meridian becomes in the development a rectangle having its meridional dimensions greatest.

2. Imagine a hollow globe formed of a mere surface of paper; suppose it actually cut by a great number of parallel planes along equidistant parallels of latitude; let also one meridian (from North Pole to South Pole, 180°) be entirely cut through. In this state let the whole be opened out into a plane from the meridian exactly opposite to the one cut through, and the previously spherical surface is converted into a number of strips of paper, each of which is part of a circular belt, with the exception of the equator, which will be straight. All points which lay on the parallel whose colatitude is u on the sphere, now lie on the arc of a circle whose radius is tan. u and length 2π sin. u; moreover, the centres of all these arcs or circular strips lie in the same straight line, viz. the central meridian. In fig. 1, let P be the North Pole, P M the central meridian, U any point in that meridian whose latitude is $90-u$. Make $\mathrm{PU}=u$, $\mathrm{UC}=\tan u$, and, with C as a centre,

Fig. 1.



describe the circular arc U Q. This arc will represent the parallel of latitude passing through U on the sphere, and the curves which cut all circles described according to this law (u variable) at right angles, are taken to represent meridian lines. To investigate the nature of this curve take another point, U′ indefinitely near to U, and let $UU' = du$, $U'C' = \tan(u+du)$, and with the centre C′ and radius C′ U′ describe the circular arc U′ Q′ indefinitely near to U Q. Let P Q Q′ be a part of one of the curves we are investigating; join C Q, C′ Q′; these lines will—being perpendicular to the circles—be tangents to the curve. Let $UCQ = 2\varphi$; $UC'Q' = 2(\varphi + d\varphi)$, then the small angle C Q C′ or the inclination of the tangents at Q and Q′ will $= 2\,d\varphi$. Now,

$$\begin{aligned} CC' &= C'U' - CU - UU' \\ &= \tan(u + du) - \tan u - du \\ &= \tan^2 u \,.\, du \end{aligned}$$

And from the triangle Q C C′

$$\tan^2 u \,.\, du \,.\, \sin 2\phi = -2 \tan u \, d\phi$$

$$\therefore\ -\tan u \,.\, du = \frac{2\,d\phi}{\sin 2\phi}$$

Integrating this equation, we have

$$\log \cos u = \log \tan \phi + \text{constant}$$

$$\therefore\ \tan \phi = w \cos u$$

Where w is a constant determining any one particular curve.

3. To determine in what point this cuts the equator, we observe that the distance of any point Q in the curve from the central meridian is $\tan u \,.\, \sin 2\varphi$; which

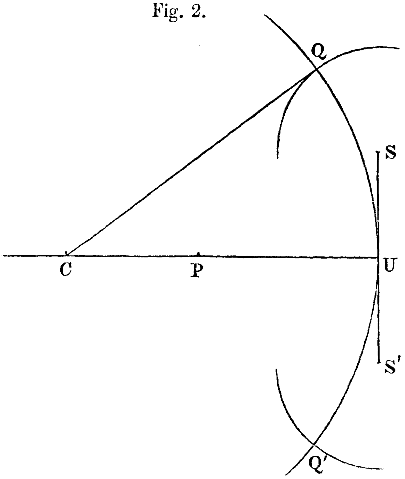
$$= \frac{2 \tan u \tan \phi}{1 + \tan^2 \phi} = \frac{2\,w \sin u}{1 + w^2 \cos^2 u}$$

and this at the equator $= 2\,w$.

Let any equatorial point whose actual longitude is $2\,w$ on the sphere be represented by a point on the developed equator at the distance $2\,w$ from the central meridian; then, remembering that in colatitude u the radius of parallel is $\sin u$, we have the following simple construction:—

4. Let P (fig. 2) be the pole, U any point in the central meridian; Q U Q′ the circular arc representing the parallel through U. Draw the tangent S U S′ perpendicular to P U through the point U: then to determine the point Q whose longitude is given (say 3° 0′) lay off U S equal to half the true length of the arc of parallel on the sphere (1° 30′, to radius $\sin u$), and with the centre S and distance S U describe a circular arc which will intersect the parallel in the required point Q. For if we

Fig. 2.



suppose $2w$ to be the longitude of the required point Q (whose colatitude is u), U S is by construction equal to $w \sin u$, and since $CU = \tan u$, the angle subtended by S U at C is

$$\tan^{-1}\left\{\frac{w \sin u}{\tan u}\right\} = \tan^{-1}(w \cos u) = \phi$$

and therefore $QCU = 2\phi$, and the distance of Q from the central meridian C U is $\tan u \sin 2\phi$, as it should be.

This very simple construction was given to me by Mr. O'Farrell, of this office, who obtained it from an ingenious process of trial and error.

The advantages of this method are that, with a remarkably facile construction, we have a map in which the meridians and parallels intersect at right angles, and the effect is pleasing to the eye. The circles for parallels are drawn in the usual manner, but the meridians, not being circular arcs, may be drawn in after the construction of the points of intersection by suitable mechanical curves.

5. The radius of curvature of the meridian whose longitude is $2w$, at the colatitude u, is readily determined. In the first figure we have $C'Q' = CQ + CC' \cos 2\phi + QQ'$. Here QQ' is the element of length of the curve, or $QQ' = ds$, also $CC' = \tan^2 u \, . \, du$; therefore we have

$$\begin{aligned} ds &= (\sec^2 u - \tan^2 u \cos 2\phi)\, du \\ &= \left(2\,(1 + w^2) \cos^2 \phi - 1\right) du \end{aligned}$$

Also $\sec^2\varphi\, d\varphi = -w\,.\sin.\,u\, du$; therefore, if ρ be the radius of curvature of the meridian, we have, by substitution,

$$\rho = \frac{1 + w^2 + w^2 \sin^2 u}{2\, w \sin u}$$

6. Let us now consider the distortion, and, for this purpose, imagine a small square described on the surface of the sphere having its sides parallel and perpendicular to the meridian. Let u and $2\,w$ define its position, and let i be the length of the side. If we differentiate the equation $\tan\varphi = w \cos u$ on the supposition of u being constant, we have

$$\sec^2 \phi\, d\phi = \cos u\, dw$$

also the length of the representation of $2\, dw$ is $2 \tan u\, d\varphi$, or

$$\sin u \cos^2 \phi\, d\,.\,2\,w$$

Hence that side of the square which is parallel to the equator will be represented by a line equal to

$$i \cos^2 \phi$$

similarly the meridional side will be represented by a line equal to

$$i \cos^2 \phi\, (1 + w^2 + w^2 \sin^2 u)$$

The square, therefore, is represented by a rectangle whose sides have the proportion

$$1 + w^2 + w^2 \sin^2 u : 1$$

and its area is increased in the proportion of

$$\frac{1 + w^2 + w^2 \sin^2 u}{(1 + w^2 \cos^2 u)^2} : 1$$

If we make this ratio equal to unity, there results the following equation—

$$w^4 \cos^4 u + 3\, w^2 \cos^2 u - 2\, w^2 = o$$

which is satisfied either by $w = o$, or by

$$w^2 \cos^4 u + 3 \cos^2 u - 2 = o$$

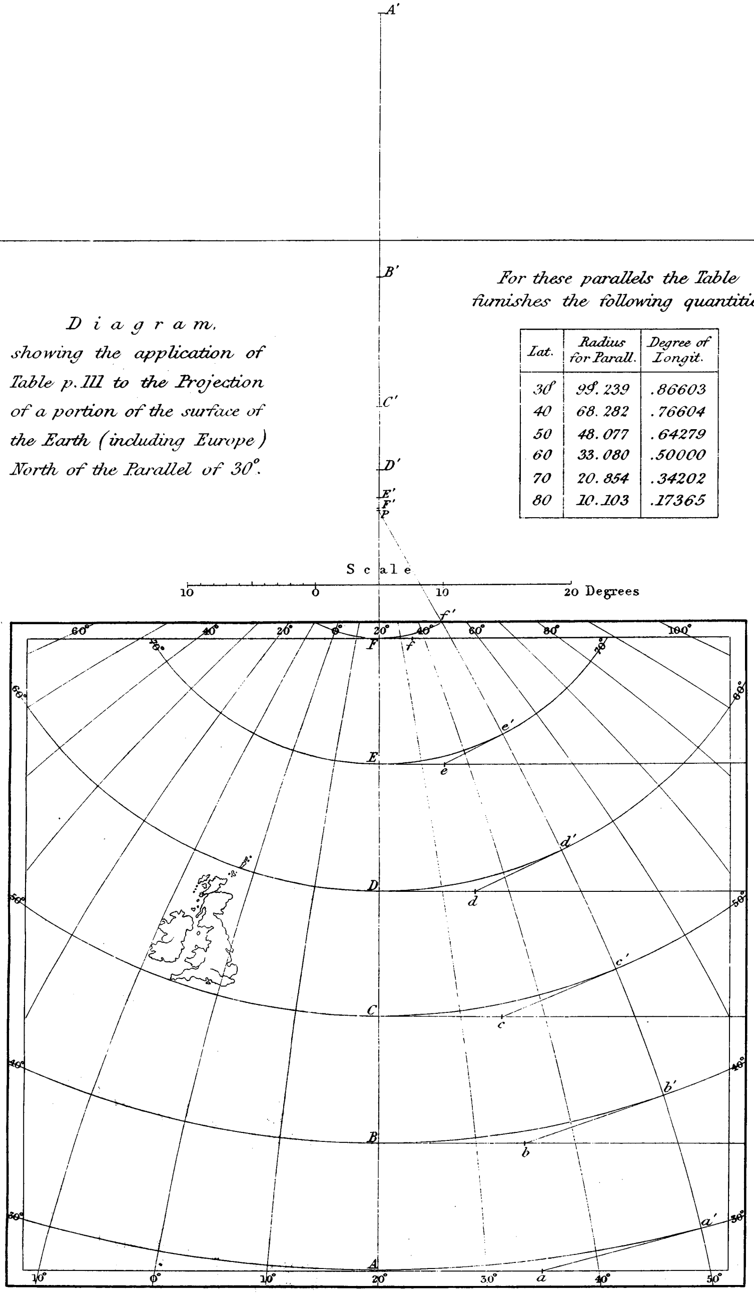
From this it appears that there is no exaggeration of area along the meridian or along the particular curve which is defined by the equation just written down. This curve crosses the central meridian at right angles in the latitude of about 54° 44′; it thence slowly inclines southward, and at 90° of longitude from the central meridian reaches 50° 26′ of latitude; at 180°, or the opposite meridian, it has reached 43° 46′. The areas of all tracts of countries lying on the north side of this curve will be diminished in the representation, and for all tracts of country south of this curve the areas will be increased in the representation.

7. This method is particularly well adapted for the construction of maps of continents, and even in the map of Asia there is very little distortion, as may be seen by an examination of the lines

Diagram,
showing the application of
Table p. III to the Projection
of a portion of the surface of
the Earth (including Europe)
North of the Parallel of 30°.

For these parallels the Table furnishes the following quantities.

Lat.	*Radius for Parall.*	*Degree of Longit.*
30°	99°.239	.86603
40	68.282	.76604
50	48.077	.64279
60	33.080	.50000
70	20.854	.34202
80	10.103	.17365



Explanation. *The line ABPB' is the Central Meridian: P the North Pole. Make* $AP = 60^\circ$, *and set off* $AB = BC = CD = DE = EF = FP = 10^\circ$ *each. The points A' (beyond the margin), B', C', D', &c. are the centres of the Circles passing through A, B, C, D, &c. respectively, and are determined by making* $AA' = 99^\circ.239$; $BB' = 68^\circ.282$; $CC' = 48^\circ.077$; $DD' = 33^\circ.080$; $EE' = 20^\circ.854$; $FF' = 10^\circ.103$, *which numbers are found opposite their corresponding Latitudes in Table p. III.*

To draw a Meridian — say that of 50° E, which is 30° of longitude from ABP. Through A, B, C, &c. draw the lines Aa, Bb, Cc, &c. perpendicular to ABP, and make $Aa = aa' = .866 \times \frac{30}{2} = 12^\circ.99$; $Bb = bb' = .766 \times 15 = 11^\circ.49$; $Cc = cc' = .643 \times 15 = 9^\circ.64$; $Dd = dd' = .500 \times 15 = 7^\circ.50$; *&c. The curve line connecting the points a', b', c' .... P is the projection of this Meridian. The same quantities serve for drawing the Meridian 30° on the left hand side of ABP; and similarly for the other Meridians.*

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J. W. Lowry.

in the accompanying figure, which is sufficient to embrace the whole of Asia. If we represent the surface of the whole globe continuously, the area of the representation is

$$(4+\pi^2)\tan^{-1}\frac{\pi}{2}+2\pi$$

which is greater than the true surface of the globe in the proportion of 8 to 5. The perimeter of the representation is equal to the circumference of the globe multiplied by $\sqrt{4+\pi^2}-1$, or 2·72.

The following Table contains the lengths of the radius for the different arcs of parallel, and also the lengths of a degree of longitude in different latitudes; the unit of length being that chosen for one degree at the equator. By the aid of this Table the construction of a projection on any scale is rendered extremely simple.

Lat.	Radius for Parallel.	Degree of Longitude.	Lat.	Radius for Parallel.	Degree of Longitude.	Lat.	Radius for Parallel.	Degree of Longitude.
°			°			°		
0	∞	1·00000	30	99·239	·86603	60	33·080	·50000
1	3282·473	·99985	31	95·356	·85717	61	31·760	·48481
2	1640·736	·99939	32	91·692	·84805	62	30·465	·46947
3	1093·268	·99863	33	88·228	·83867	63	29·194	·45399
4	819·368	·99756	34	84·944	·82904	64	27·945	·43837
5	654·894	·99619	35	81·827	·81915	65	26·717	·42262
6	545·133	·99452	36	78·861	·80902	66	25·510	·40674
7	466·637	·99255	37	76·034	·79864	67	24·321	·39073
8	407·681	·99027	38	73·335	·78801	68	23·149	·37461
9	361·751	·98769	39	70·754	·77715	69	21·994	·35837
10	324·940	·98481	40	68·282	·76604	70	20·854	·34202
11	294·761	·98163	41	65·911	·75471	71	19·729	·32557
12	269·556	·97815	42	63·633	·74314	72	18·617	·30902
13	248·175	·97437	43	61·442	·73135	73	17·517	·29237
14	229·801	·97030	44	59·331	·71934	74	16·429	·27564
15	213·831	·96593	45	57·296	·70711	75	15·352	·25882
16	199·814	·96126	46	55·330	·69466	76	14·285	·24192
17	187·406	·95630	47	53·429	·68200	77	13·228	·22495
18	176·338	·95106	48	51·589	·66913	78	12·179	·20791
19	166·399	·94552	49	49·806	·65606	79	11·137	·19081
20	157·419	·93969	50	48·077	·64279	80	10·103	·17365
21	149·261	·93358	51	46·397	·62932	81	9·075	·15643
22	141·812	·92718	52	44·764	·61566	82	8·052	·13917
23	134·980	·92050	53	43·175	·60181	83	7·035	·12187
24	128·688	·91355	54	41·628	·58779	84	6·022	·10453
25	122·871	·90631	55	40·119	·57358	85	5·013	·08716
26	117·474	·89879	56	38·646	·55919	86	4·007	·06976
27	112·449	·89101	57	37·208	·54464	87	3·003	·05234
28	107·758	·88295	58	35·802	·52992	88	2·001	·03490
29	103·364	·87462	59	34·427	·51504	89	1·000	·01745
30	99·239	·86603	60	33·080	·50000	90	0·000	·00000

Degree of Equator = Degree of Meridian = 1.
Radius of sphere = 57·2958.